

**Semester 1 Examination 2012**

**Question/Answer Booklet**

**MATHEMATICS 3CD**

**Section One:**

**Calculator-free**

Name of Student: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Marking key\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: 5 minutes

Working time for this section: 50 minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer Booklet

Formula Sheet

***To be provided by the student***

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler,

 highlighters

Special items: nil

**Important note to students**

No other items may be used in this section of the examination. It is **your** responsibility to ensure

that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time(minutes) | Marks available | Percentage of exam |
| Section OneCalculator-free | 6 | 6 | 50 | 50 |  |
| Section TwoCalculator-assumed | 12 | 12 | 100 | 100 |  |

|  |  |  |
| --- | --- | --- |
| Total | 150 | 100 |

**Instructions to students**

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

**Section One: Calculator-free (50 marks)**

This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Question 1 (8 marks)**

(a) Given the following Venn Diagram showing events A and B



Determine x if

1. A and B are mutually exclusive (1)
2. A and B are independent. (The condition from (i) does not necessarily hold)

(3)

|  |
| --- |
| **Solution** |
| 1. A and B are mutually exclusive if

So x = 01.
 |
| **Specific behaviours** |
| ✓ factorises denominator✓ critical values of -3,-1,1✓✓ correct intervals |

(b) The functions and are defined as follows

and 

(i) Determine expressions for . (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓ I mark for each |

(ii) Determine the range of . (1)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ or X |

(iii) Determine the domain of . (1)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ or X |

**Question 2 (9 marks)**

(a) Differentiate the following with respect to *x.*

1.  (express in simplest form) (3)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓ uses quotient rule correctly✓ simplifies to  |

1.  (do not simplify) (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓ uses product rule and differentiate correctly |

(b) Sketch the graph of the derivative function for on the axes below. (2)

 

 

|  |
| --- |
| **Solution** |
| As shown on graph above |
| **Specific behaviours** |
| ✓ shape✓ all below x-axis |

**Question 2 (continued)**

(c) Given the derivative function, sketch the graph of the function. (2)

 

 

|  |
| --- |
| **Solution** |
| As shown above |
| **Specific behaviours** |
| ✓ shape✓ turning point and within boundary |

**Question 3 (7 marks)**

(a) It is claimed that the tangent to the curve  at the point where *x*=1 passes through the point (3, 8). Is this claim valid? Justify your answer. (5)

|  |
| --- |
| **Solution** |
| Equation of tangent at (1,-2) is   Substitute (3, 8) into equation   XClaim is not valid as the tangent at (1,-2) to the curve does not pass through (3, 8) |
| **Specific behaviours** |
| ✓y = -2 when x = 1✓ gradient function✓ gradient at (1,-2)✓ equation of tangent✓ substitute (3, 8) and states claim is not valid |

(b) Two identical coins are tossed together, and the outcome is recorded. After a large number of trials it is observed that the probability that both coins land showing heads is 0.36.

What is the probability that both coins land showing tails? (2)

|  |
| --- |
| **Solution** |
| P(1 Tail) = 0.4P(2Tails)=0.4X0.4 = 0.16 |
| **Specific behaviours** |
| ✓ probability of 1 tail✓ correct answer of 0.16 |

**Question 4 (8 marks)**

The volume of a certain rectangular box is given by the equation .

(i) If the height of the box is units, determine an algebraic expression for the area of the base of the box. (3)

|  |
| --- |
| **Solution** |
| Area of base =  |
| **Specific behaviours** |
| ✓✓ uses Long Division✓ correct answer of  |

(ii) Calculate the value of *x* for which the volume is a maximum. (5)

|  |
| --- |
| **Solution** |
|  or When , When , Value of x when volume is a maximum is  |
| **Specific behaviours** |
| ✓ ✓ ✓ Equates first derivative to zero , factorises and solve for x values✓ second derivative test for max and min✓ states  when volume is max |

**Question 5 (10 marks)**

(a) A spherical balloon is being deflated in such a way that the volume is decreasing at a constant rate of 120cm3/sec. At time *t* (seconds), the radius of the balloon is *r* cm.

Find the rate of change of the surface area when the volume is cm3. (5)

|  |
| --- |
| **Solution** |
| When , , , , =, = = =  cm2/sec |
| **Specific behaviours** |
| ✓ ✓ ✓ ✓ ✓  |

**Question 5 (continued)**

(b) Events *A* and *B* are such ,  and 

1. Show that event *A* and *B* are **NOT** mutually exclusive. (3)

|  |
| --- |
| **Solution** |
|  OR  As  A and B are NOT mutually exclusive  A and B are NOT mutually exclusive  |
| **Specific behaviours** |
| ✓  value✓ value✓ as they are not equal, concludes A and B are not M.E. |

1. Hence find . (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses ✓ correct value for  |

**Question 6 (8 marks)**

(a) Determine  (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expands  correctly✓ integrates each term correctly |

(b) Determine  (2)

|  |
| --- |
| **Solution** |
| = =  |
| **Specific behaviours** |
| ✓ integrates to obtain ✓ simplify k to  |

(c)  is defined such that  and 

Find

1. . (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  ✓ ✓ or X |

**Question 6 (continued)**

(c) (ii)  (2)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓✓ |